HOW LONG IS THE COASTLINE OF GIBRALTAR? FRACTAL DIMENSIONS AND SOME ECOLOGICAL AND CONSERVATION IMPLICATIONS.

Darren Fa / Marine Section, Gibraltar Ornithological & Natural History Society and Gibraltar Museum.

Resumen

La utilidad del concepto matemático de fractales para el estudio de la ecología litoral se estudió teóricamente tras ilustrar el uso del método para medir la zona costera de Gibraltar. Las diferencias en riqueza de especies marinas entre las costas este y oeste de Gibraltar se utiliza como ejemplo.

Abstract

The usefulness of the mathematical concept of fractals as a tool for the study of coastal ecology is discussed after a consideration of fractal principles illustrated by reference to the measurement of the length of the coastline of Gibraltar. The differences in species richness between the east and west coasts of Gibraltar are used as example of the application of fractals in ecology.

This apparently trivial question is analogous to one posed by Benoit Mandelbrot in his article entitled 'How long is the coast of Britain? It was first asked by Lewis Fry Richardson (see Mandelbrot, 1967) when he observed that estimates of the common frontiers of the national borders of Spain, Portugal, Belgium and the Netherlands varied by up to twenty per cent depending on the encyclopaedia used. Mandelbrot (1967) proposed the surprising answer; that the estimated length of any coastline depends on the length of the 'yardstick' used to measure it and as this decreases in size the length of the coastline will tend towards infinity!

This statement obviously needs to be qualified, and Mandelbrot did so by showing that coastlines are fractal boundaries. But what, then, is a fractal?

A fractal can be defined as a geometrical object with a non-integer dimension (Frontier 1987). Used as we are to classifying objects as occupying zero dimensions (a point), one dimension (a line or curve), two dimensions (a surface) or three dimensions (a volume), the concept of a dimension that is not a whole number can cause some consternation. Nevertheless, it was the inability of Euclidean geometry to capture the essence of irregular shapes that led Mandelbrot to the seeming impossibility of fractional or fractal dimensions. Consider a particle moving randomly in a plane. As time increases, its path will become more and more complex and the particle will progressively visit more and more points on the plane. Ultimately, then, given an infinite timespan, the particle would have visited every point in the two dimensional space. In this extreme case the topological dimension of the path would be one (a line), but it is actually covering two dimensions. The fractal dimension (δ) is therefore an attempt to quantify various levels between the one-dimensional straight line and the two-dimensional plane. In this context, it could be described as the 'space -filling' capacity of the path. The more complex the path, the more the value of (approaches two (See Table 1).

Topological dimension	Line Segments	Fractal dimension
1		1.00
1		1.02
1		1.25
1		1.45

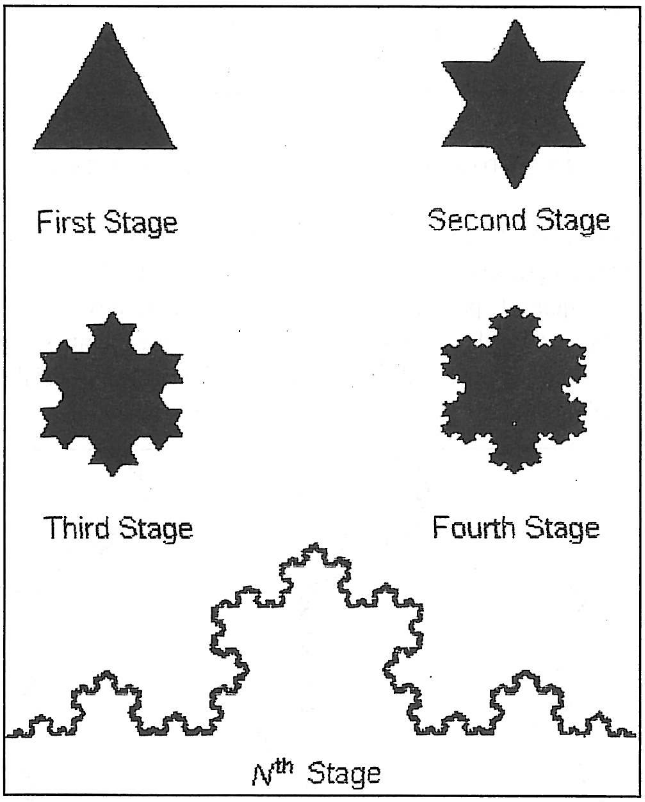
(Modified from Kaye 1994)

Table 1. Extending the topological dimension to a fractional dimension

(This relationship is just as applicable to dimensions between 0 and 1 and also between 2 and 3 but these do not concern us here - Note that the topological dimension remains unchanged).

As already mentioned, many people find the concept of a fractional (i.e. non-integer) dimension intuitively difficult to comprehend. However, one must realise that what a fractal dimension gives us is a tool for analizing the surprisingly common situations where the simple lines and surfaces of classical Euclidean geometry are found to be lacking. It is rather analogous to stating that the average family has 2.2 children. Few people have a problem with this yet no-one tries to visualize what 0.2 of a child looks like.

One of the properties that characterises mathematical fractals is self-similarity, in other words, the boundary will look exactly the same at all levels of magnification. I shall attempt to explain this concept using Koch's triadic island of the Nth order (Mandelbrot 1977). This fascinating mathematical curve has a rather perplexing property; that of an infinite perimeter enclosing a finite area.

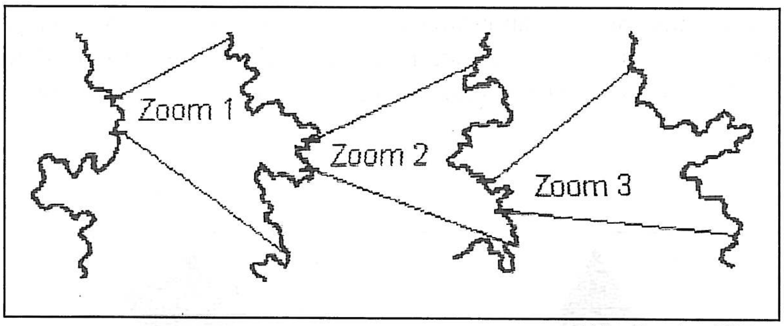


(After Gleick, 1987)

Fig. 1. The Koch Curve

This curve is constructed by starting with an equilateral triangle with sides of length 1 and repeatedly adding a new triangle one-third the original size to the middle of each side. The length of the boundary steadily increases as new triangles are added in the following sequence; 3 x 4/3 x 4/3 x 4/3 x ...and so on to infinity. Nevertheless, the area encompassed will always remain less than the area of a circle drawn around the original triangle - a finite area bounded by an infinitely long line.

But are boundaries in natural systems fractals? In the mathematical sense, no. True fractals are abstractions, unlike real-world phenomena, which can only operate over a finite range of scales. Nevertheless, Mandelbrot (1967) showed that whilst natural boundaries would not satisfy the strict prerequisite for a fractal curve to exhibit exact self-similarity at all scales, they could be statistically self-similar, i.e. on the average, subsections of the selected boundary will, on repeated magnification, look similar to each other.



(After Frontier, 1987)

Fig. 2 Fractal Dimension of a Rocky Coastline: Statistical Self-Similarity

A relatively straightforward (if labour-intensive) way of calculating the fractal dimension of a coastline is to continually recalculate its perimeter (P) using progressively smaller step lengths. A set of dividers set at a determined step length (λ) is used and the number of steps (n) required to provide an estimate of the coastline are counted. The resulting number of steps is merely an approximation of the true length, as the dividers will skip over irregularities smaller than λ. When repeated at smaller and smaller values of λ, more and more of the surface detail will be included, and the estimate of the coastline will grow longer and longer, approaching infinity. Hence the answer to the original question.

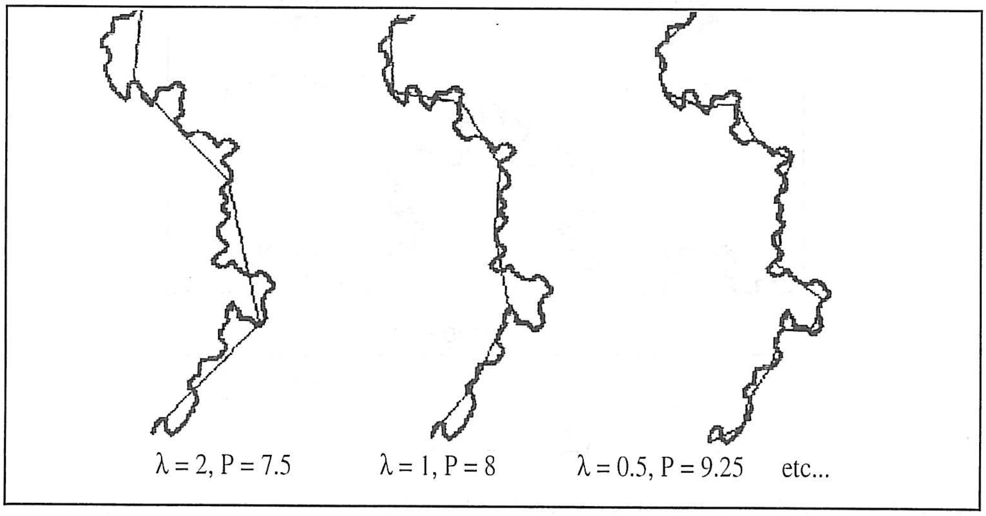


Fig. 3 Using the step technique to estimate the perimeter of a coastline
(Note that the value of P will continue to grow as λ decreases).

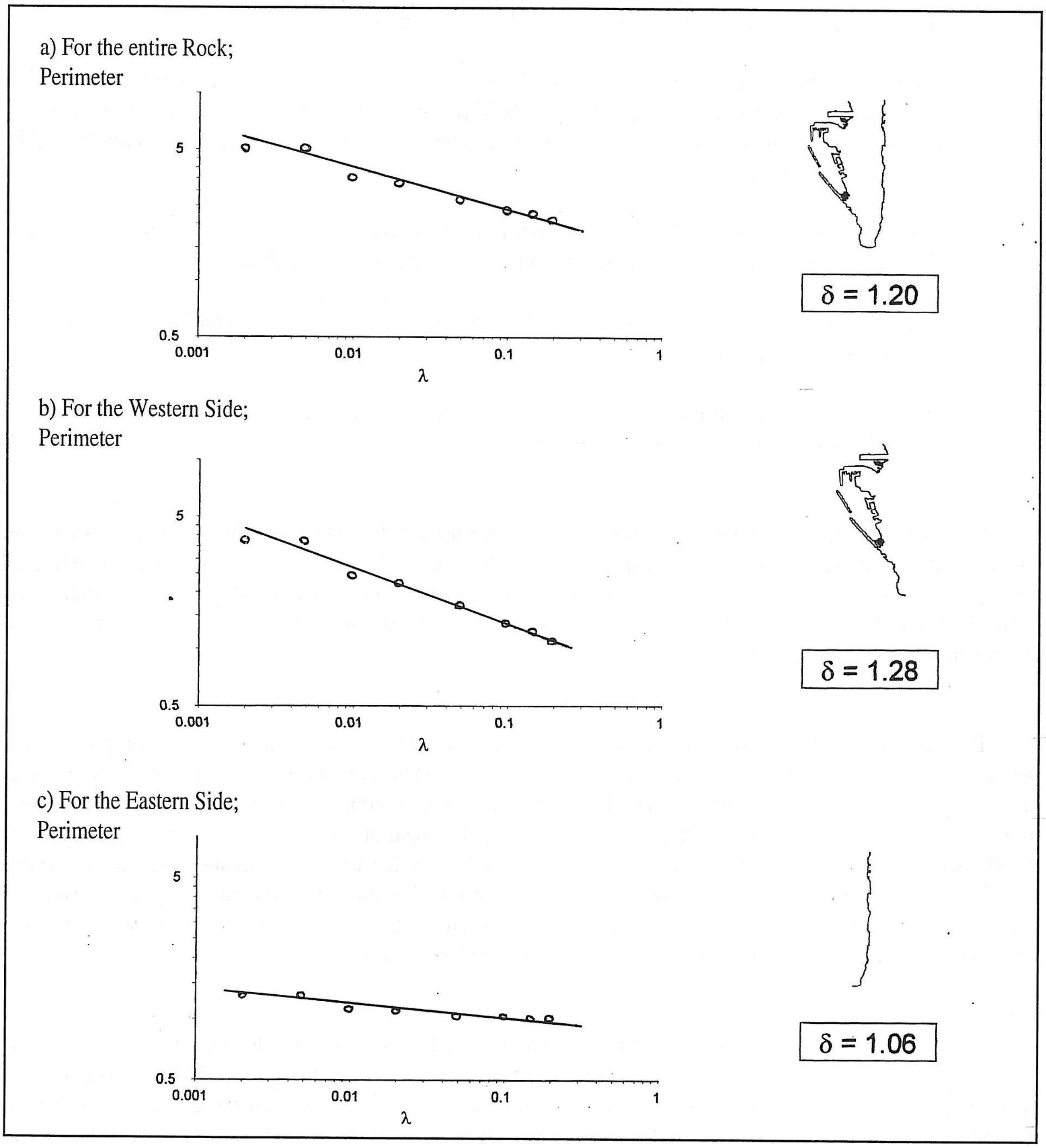


Fig. 4.- Richardson Plots For Gibraltar's Coastline

Blanket application of a fractal measurement technique can sometimes cause the suppression of important data, as illustrated by the case of Gibraltar's coastline, which has a more rugged structure on the western side than on the east. (following an example by Kaye 1994).
Note also that the detached mole was not included in the calculation of λ.

The actual value of the fractal dimension can be obtained by following these steps:

I) Estimate the length of the coastline by using a range of values of λ, provided that λ does not exceed a third the value of the maximum projected length of the profile (F_0) - the Maximum Feret's Diameter (See Kaye 1994 for reasons). It is usually also recommended that this be repeated various times using different starting points for each value of λ, as small variations can occur.

i) Normalize the estimates for P and λ by dividing them by F_0. This has the effect of converting these quantities into a dimensionless form which allows for direct comparisons with other fractal boundaries.

ii)These values of P and λ are plotted on a log-log or Richardson plot and the slope (m) of the line joining the points is calculated by linear regression.

iii) Add 1 (the topological dimension of the line) to the modulus of m ($|m|$) to obtain the fractal dimension (δ) (See Kaye, 1994, for alternative ways of calculating δ).

It becomes then relatively simple to calculate the fractal dimension of Gibraltar's coastline and fig. 4(a) shows the Richardson plot obtained by plotting the total perimeter against λ. It is a straight line with $|m| = 0.20$. Consequently the fractal dimension (δ) of the Rocks' coast is 1.20, and can be taken to be a measure of the overall complexity of the coastline. (The 'alternate step' method of Kaye 1994 was used, which involves taking alternate inswings and outswings with the dividers, and provides a more balanced estimation of δ).

How then, can this abstract mathematical concept be of use to ecologists and conservationists? A simple possibility is outlined below. Fig. 4(a) showed the fractal dimension of Gibraltar's coastline as calculated using the alternate step method. However, it will be seen that the complexity of the Rock's coastline is not uniform; the western side is much more complex, as evidenced by the higher number of jetties, moles, and housing developments. Note that when the fractal dimension is calculated for each side, the western side has a fractal dimension of 1.28, as shown by the greater slope of the regression line (Fig. 4(b)), whereas the eastern side has a low fractal dimension of 1.06 (Fig. 4(c)). This illustrates the potential danger of aggregating fractal surfaces if they are clearly different, as important information may be suppressed, but allows us a measure that can help to suggest reasons for variations in biodiversity found around the coast.

The higher fractal dimension of the western coast indicates a higher space-filling ability than the eastern side. This increased complexity means that the fluxes and exchanges of energy, matter and information are enhanced (Frontier 1987), creating a particular type of ecosystem characterized by an 'area of interpenetration' which in this case is the littoral zone. This situation is further improved by fractal morphologies present in accompanying physical systems such as water turbulence and eddy effects which are directly linked to the complexity of the coastline. The western side of the Rock is therefore not only richer in rocky shore habitats in terms of exposure, orientation, etc., but also experiences a greater variety of small-scale water movements that together create microclimates, enriching localized habitat diversity which in turn affects species diversity.

Limnologists have for a long time been aware of a relationship between the morphology of ponds and lakes, and such biological properties such as overall productivity, etc. (Hutchinson 1957; Ryder 1965; Wetzel 1975; Adams and Oliver 1977). Lake morphology, as well as the 'morphoedaphic index', have been expressed in terms of a ratio between water volume and coastline length. However, as we have already seen, the notion of coastline length is in reality a non-concept and needs to be linked to the scale used. Consequently, it would seem sensible to propose that it is the fractal dimension which needs to be correlated with ecosystem properties instead of the length/volume ratio.

If our assumptions are correct, this increased complexity on the western side should be evidenced by a higher species richness and variability on the western side than on the eastern side. This notion was explored in a very superficial manner by comparing littoral datasets from a number of rocky sites on both the western and eastern shorelines (Fa 1990 & this study). These were censused using a fixed belt transect technique, sampling every 0.25 metres in height using a 1x0.25m quadrat (see Fa 1990 for further details). All the sites faced out to sea, so the full range of possible conditions (particularly on the western side) was not sampled, but it was hoped that by keeping orientations and substrate types the same, differences between samples would be restricted to those caused by differences in water movements and wave action.

Table 2 shows the differences between sides in terms of overall and mean species richness. The total aggregate species richness for each side is higher for the western side (41 species) than for the eastern side (32 species). This increase is not due to any one particular site (the mean species richness values for each side are practically the same), but rather to an increased variability as evidenced by the higher standard deviation (6.23) shown by the western side sites. Conversely, a low standard deviation value of 1.92 for the eastern side sites indicates little variability between these sites. This is in keeping with our predictions.

	Species Richness		
	Aggregate	Mean	Standard Deviation
Western side	41	21.75	6.23
Eastern side	32	21.80	1.92

Table 2. Comparisons of Species Richness for West and East Coast Sites

Moreover, the sublittoral fringe was also sampled at four of these sites (two western, two eastern). This area is considered to be highly productive and also is not exposed at low tide, thus freeing the organisms that inhabit this zone from needing elaborate physiological or behavioural desiccation-avoidance strategies, an important factor which restricts the species pool that can be found on rocky shores. Although the low number of replicates precludes any form of statistical analysis, the total number of species for the western sites increased by an overall of 57, making the aggregate total for the western side 98 species, whereas only 20 new species were encountered on the eastern sites, raising its aggregate to 52 species. The higher diversities on the western side are not restricted to littoral areas, but are also found in other marine communities (A. Menez, E. Shaw, pers. comm.).

Although by no means an exhaustive analysis, this exercise serves to illustrate the proposal that fractal morphologies could and should be correlated to ecosystem properties.

This is not to say that fractal morphologies should in any way be used as a substitute for biotic and abiotic factors in the structuring of communities, but rather that this complexity modifies these factors to such an extent that the habitat and consequently the specific diversity of these sites is enhanced (Palmer 1992), an important consideration in conservation ecology.

However, even though we have established that the length of a coastline, as measured from a map, is arbitrary and depends on the step length being used, certain limitations of the model need to be mentioned. As already stated, most natural and biological objects are not true fractals; they are produced by processes that act over a finite range of scales. Thus many natural objects show variation of δ with scale, i.e. a shift in the apparent dimension of the object. These are usually evidenced by sharp truncations in the line drawn on a Richardson plot indicating other fractal dimensions beyond certain levels of examination and emphasises the need for any estimate of δ to be accompanied by the range of step values over which it holds.

Within each range of constant δ, it may be that a (possibly complex) self-similar generating process is acting. However, once a shift in δ is reached, one can no longer make extrapolations and a new generating process should be suspected. Rather than obscuring the issue, these 'transition zones' (Mandelbrot 1977) are of particular interest, as they indicate points at which the spatial structure of the system changes. Therefore, not only do the fractal dimensions allow us a useful measure which can be correlated to other biological processes, but moreover can serve to illustrate the relative importance of different processes at particular scales, allowing the detection of functional hierarchies (Sugihara & May, 1990) e.g. for ecologists they might correspond to changes in levels of resource use and community structure. This phenomenon can be best explained by analogy to a flat road surface 1km long. Whilst it seems smooth to us, and to other organisms at similar spatial scales, (as would be shown by taking steps of size 20m, 10m, 5m, 1m, etc.) there will come a point where the step length will be small enough to suddenly take into account the varying rugosities and crevices in the tarmac. At this level, then, the fractal dimension would suddenly increase, indicating an increased roughness, one that would certainly not be of trivial value to say, an ant! Alternatively, these truncations could indicate points where the investment of energy and resources by an organism, e.g. a branching coral, are no longer justified by returns (e.g. the devil's comb of Villermaux et. al. 1986 a,b), or no longer can be maintained due to the limitations of biological material itself (See Bradbury et. al., 1984 and Krummel et. al., 1987 for examples of hierarchical scaling in a coral reef and a deciduous woodland respectively).

It is clear, then, that there is something deeper at stake here than the trivial practicalities involved in accurate measuring; a coastline, a treetrunk, a branching coral all have literally longer and longer boundaries as λ tends to zero, and this has implications regarding the way these habitats look to creatures of different sizes (Sugihara & May 1990; see also Gee & Warwick 1994).

These applications can be extended to the fractal characteristics of distributions such as the scattering of species, habitats or resources in space, allowing us a measure of complex order which has, in the past, been so difficult to encapsulate effectively. Other possible applications as outlined by Frontier 1987 and Sugihara & May 1990 include using the relationship between fractals and modified brownian dynamics to investigate the fractal properties of a time series of population values or the analysis of species' excursions. Conservationists might include notions of boundary complexity in the design of nature reserves. Moreover, and in a different vein, we have random iteration algorithms (Barnsley et. al. 1986), which by repeated iteration of a few relatively simple equations produce extremely complex fractal patterns that are remarkably similar to natural objects such as fern leaves, the equivalent euclidean description for which might involve a polynomial with thousands of fitted parameters. The implications of such a small amount of information producing such complex natural forms may have important consequences for areas like genetics.

Thus the original, apparently trivial question has turned out to be much more complex than at first imagined, and its solution a possible enlightenment, providing us with a new, fresh insight into the structure and workings of individuals, populations and communities. The universality of fractal systems and their potential for the economical description of natural patterns reflects a ubiquitous property of nature. They promise much in the way of investigations involving scale and hierarchical organization, a perspective that has till relatively recently been lacking in the biological sciences.

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